FFT Laboratory Experiments for the Agilent 54600 Series Oscilloscopes and Agilent 54657A/54658A Measurement Storage Modules

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Introduction:

This document provides a set of elementary experiments which illustrate the capabilities and limitations of employing the Agilent 54657A Measurement Storage Module for frequency domain analysis. In particular, these experiments are designed to illustrate the relationship between frequency resolution, effective signal sampling rate, spectral leakage, windowing and aliasing. It is assumed that the reader is familiar with the basic operations of the Agilent 54657A module which are described in the Agilent 54657A User Guide. Refer to the Product Note 54600-4 for additional information about the use of the Module for frequency domain analysis.

The FFT (Fast Fourier Transform) is a convenient and powerful tool for performing frequency domain analysis on a variety of signals. Since the FFT operates on discrete samples of a signal, it is important to understand the relationships between the signal sampling rate, the continuous-time Fourier Transform and the Discrete Fourier Transform (DFT).[1]

Sampling Theory

Sampling theory provides the mathematical foundations for analyzing continuous-time signals with Digital Signal Processing (DSP) methods. The ideal sampler shown in Figure 1. 1 is an important conceptual tool for integrating the continuous-time and discrete domains. Many DSP texts, such as (Reference 1-3.) use this approach when discussing sampling theory concepts.

Figure 1.2 illustrates how the ideal sampler may be used to explain the link between the Fourier Transform and the DFT. The most important conclusion to draw from Figure 1.2 is that the sampling process results in a periodic Fourier Transform as illustrated in part (d) of Figure 1.2. For finite duration signals, the DFT is obtained by taking N equally spaced samples of this periodic Fourier Transform (see Figure 1.3).

We have seen that, in effect, the DFT produces N equally spaced samples from the Fourier Transform of the original signal. In other words, the spectrum of the original signal x(t) is sampled at the frequency values

$$fk = \frac{k}{NT} Hz \quad \text{for k= 0,1,2,\ldots,N-1}$$

A frequency resolution of 1/NT Hz is the best that one could hope to achieve with an N point FFT since each frequency sample is spaced 1/NT Hz apart. Based on the expressions above, it can be seen that increasing T (reducing the sampling rate) leads to improved frequency resolution. However, in order to avoid aliasing, the sampling rate must not be reduced to less than input signal’s Nyquist rate. Figure 1.4 graphically illustrates the affect that the sampling rate has on spectral resolution.

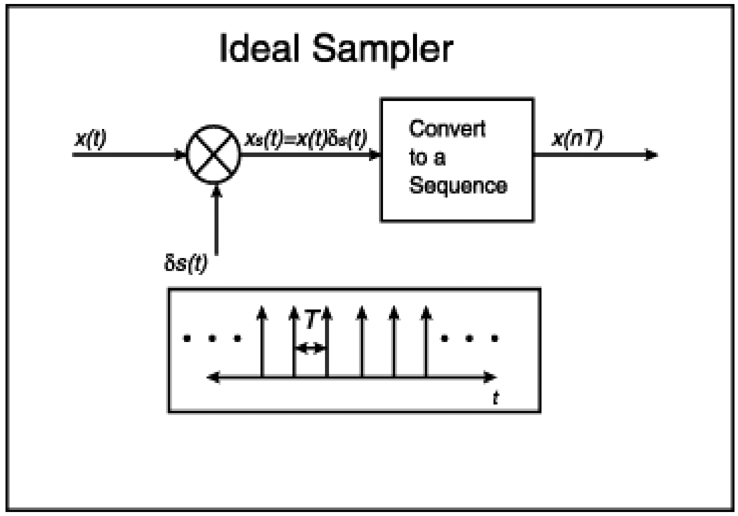

Figure 1.1

The ideal sampler is a conceptual tool for establishing a link between the continuous-time Fourier Transform and the DFT. The input to the ideal sampler is *x(t)* and the output consists of the sample values *x(nT).* The "analog" sampled signal, $x_s(t)$, and its Fourier Transform, $X_s(f)$ are shown in Figure 1.2.

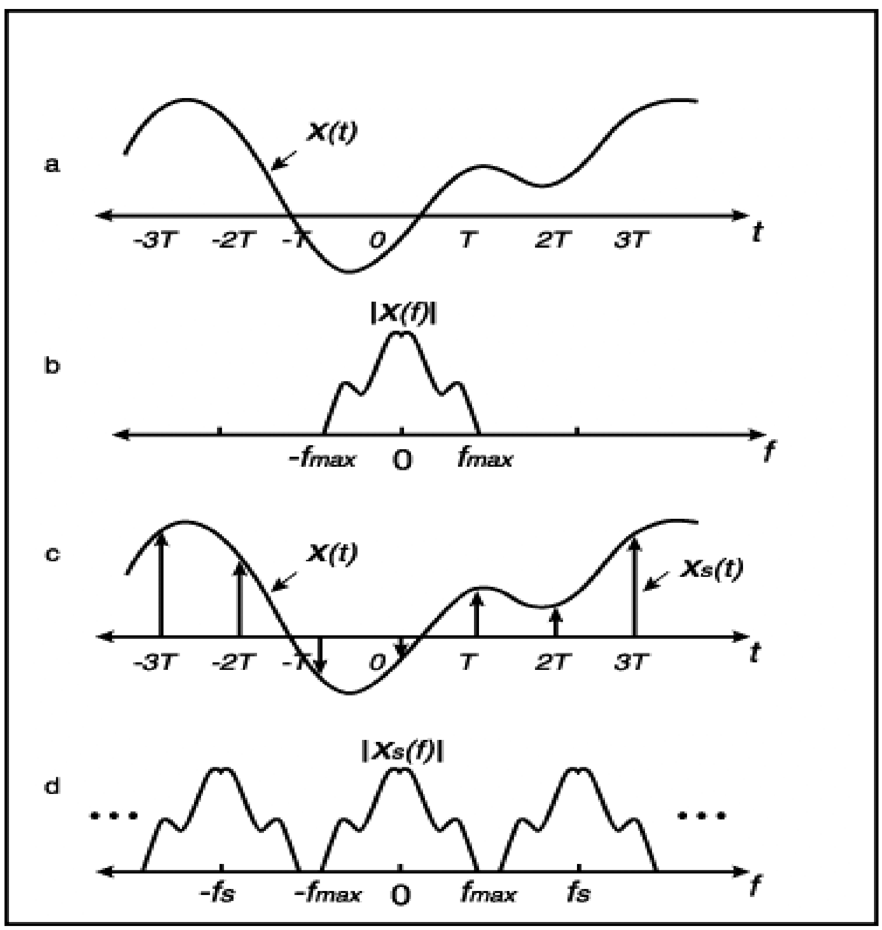

Figure 1.2

a) The original continuous-time signal, x(t).

b) **The Fourier Transform of *x(t).***

c) **The signal $x_s(t)$ (See Figure 1.1). This "conceptual" signal consists of an impulse train with each impulse weighted by x(nT) and spaced *T* seconds apart. Since $x_s(t)$ is technically a continuous-time signal, it has a Fourier Transform $X_s(f)$, which is illustrated in part (d).**

d) **The Fourier Transform of $x_s(t)$ provides an important conceptual link between the Fourier Transform of the signal of interest x(t) and the DFT of the sampled signal x(nT). Notice that $X_s(f)$ is formed by first multiplying X(*f*) by a constant value 1/T and then replicating X(*f*)/T at intervals spaced f_s=1/T apart (f_s is the effective sampling rate). If the sampling rate is not sufficiently large, then f_s will not be large enough to insure that the replicas of X(*f*)/T do not overlap. Aliasing occurs when the replicas of X(*f*)/T overlap.**

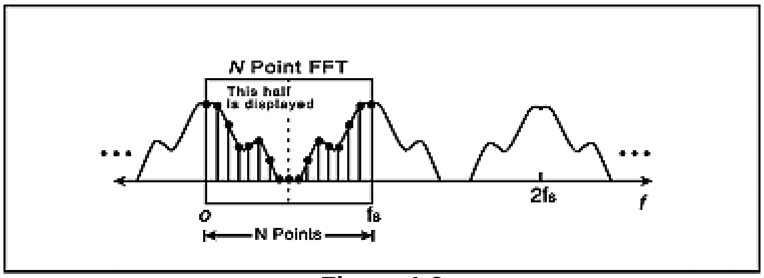

Figure 1.3

The DFT of a finite duration discrete signal *x(nT)* is obtained by sampling the spectrum of $x_s(t)$ (see Figure 1.1). These frequency samples are equally spaced over the frequency range 0 to f_s Hz. It can be shown that for real signals, the DFT will always possess the "folding" symmetry property which is illustrated above. Therefore, it is customary to only display the first half (the portion to the left of the dashed vertical line) of the FFT. The Agilent 54657A module follows this convention and thus the maximum frequency span for the FFT display is equal to $f_s/2$.

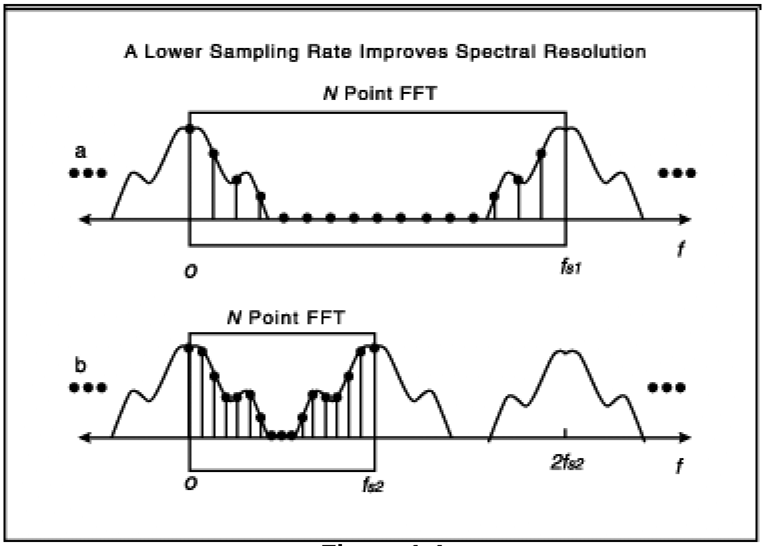

Figure 1.4

This figure graphically illustrates the effect that the sampling rate has on spectral resolution. Parts (a) and (b) depict the frequency samples obtained by using an N point FFT and sampling rates of f_{s1} and f_{s2} respectively. Note that the sampling rate used in part (a) is twice that of part (b). By lowering the sampling rate (yet still avoiding aliasing) we obtain more samples of the "interesting" region of the spectrum. Note that part (a) has fewer nonzero samples of the spectrum than does part (b).

The Effects of Windowing

Since the DFT requires a finite length input signal, an "on-going" signal must be truncated before FFT computations take place [2]. The truncation process is achieved by overlapping the input sequence x[n] with a finite length window w[n] and performing a point-by-point multiplication as illustrated in Figure 1.5. Thus, the FFT is computed on the signal y[n] which is given by

$$y[n] = x[nlw[n]$$

The windowing processes introduces a loss in spectral resolution and an effect known as spectral leakage. In general, the choice of window function involves a tradeoff between these two effects. That is, windows with better frequency resolution capabilities do not, in general, perform as well in terms of spectral leakage and vice-versa.

A wide variety of window options are available for DSP applications. In these experiments, we will employ the rectangular window, the Hanning window and the Flattop window. The rectangular and Hanning windows are illustrated in Figure 1.5. The effects of windowing are well illustrated by considering a simple sinusoid. Figure 1.6 part (a) depicts the familiar Fourier transform (magnitude, f>0) of a sinusoid signal. Of course, the spectral content of a sinusoid is represented by an impulse function concentrated at the sinusoid's fundamental frequency f_0. In order to perform an FFT analysis, we first multiply samples of the input signal by the window function. Recall that the Fourier Transform of two signals multiplied in the time domain is given by convolving the transform of each signal in the frequency domain. In general, convolution has a "smoothing" and "spreading" effect which results in spectral leakage and a loss in resolution. Figure 1.6 illustrates the frequency domain effects of windowing.

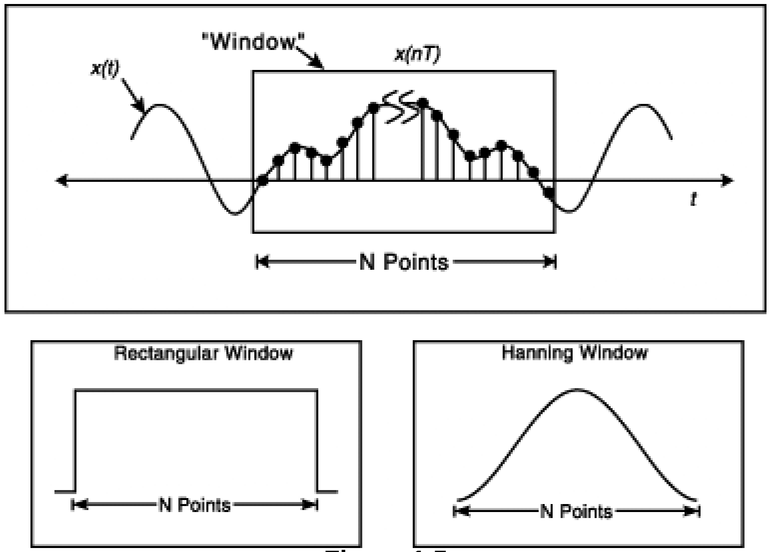

Figure 1.5

An ongoing signal is converted to a finite number of samples by the use of a window. Every sample in the box shown above is multiplied (point-by-point) by a window function. Examples of window functions include the rectangular and Hanning windows which are shown above.

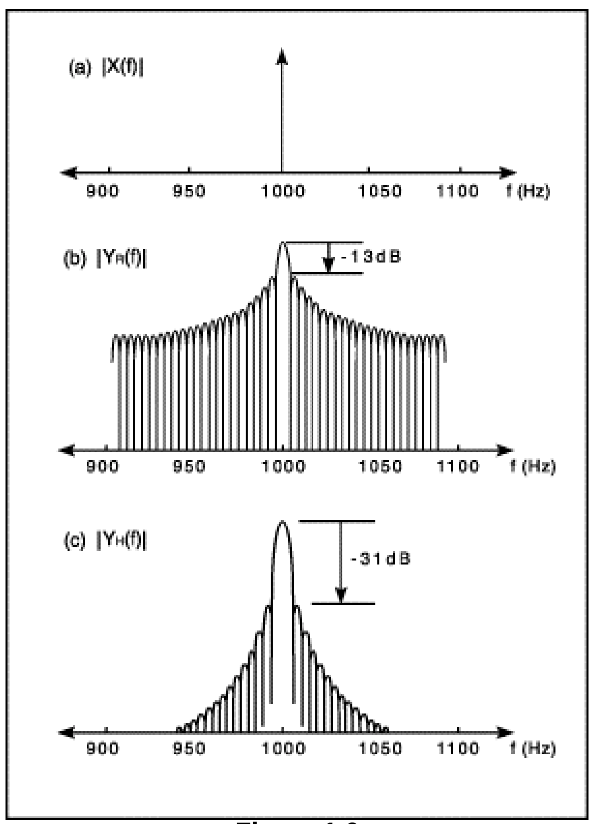

Figure 1.6

a) Fourier Transform (Magnitude) of a Sinusoid.

b) The 1024 point DFT[3] using the rectangular window. Note the poor spectral leakage property for this window. However, since spectral resolution can be considered to be a function of the "main lobe" width, the rectangular window can, in some cases,

be used to resolve closely spaced frequency components. In general, however, the large spectral leakage associated with the rectangular window makes it a less desirable choice.

c) **The 1024 point DFT using the Hanning window. By avoiding an abrupt truncation of the time-domain signal, the Gibbs effect is reduced with the result being improved spectral leakage properties. The main lobe width of the Hanning window is wider than that of the rectangular window and thus the spectral resolution is reduced. The Hanning window is popular because it achieves a good balance between spectral leakage and resolution.**

[1] An FFT is a computationally efficient method for computing the DFT.

[2]The Agilent 54657A & Agilent 54658A Measurement Storage Modules use a fixed, 1024 point FFT. The time duration of the window function is determined by the Time/Div control. By lowering the effective sampling rate (using the *Time /div* control), the window used to truncate the signal has a larger time duration.

[3]The plots shown in parts (b) and (c) are more accurately described as the Discrete-Time Fourier Transforms (DTFT) of a sinusoidal signal using a 1024 point rectangular window (part b) and a 1024 point Hanning window (part c). The plots were obtained by zero-padding the 1024 point signal to 16384 points. See (Reference 3.) for details.